

Acceleration of relativistic electrons via drift-resonant interaction with toroidal-mode Pc-5 ULF oscillations

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Abstract. There has been increasing evidence that Pc-5 ULF oscillations play a fundamental role in the dynamics of outer zone electrons. In this work we examine the adiabatic response of electrons to toroidal-mode Pc-5 field line resonances using a simplified magnetic field model. We find that electrons can be adiabatically accelerated through a drift-resonant interaction with the waves, and present expressions describing the resonance condition and half-width for resonant interaction. The presence of magnetospheric convection electric fields is seen to increase the rate of resonant energization, and allow bulk acceleration of radiation belt electrons. Conditions leading to the greatest rate of acceleration in the proposed mechanism, a nonaxisymmetric magnetic field, superimposed toroidal oscillations, and strong convection electric fields, are likely to prevail during storms associated with high solar wind speeds.

Introduction

Relativistic electron fluxes have been shown to vary over a wide range of time scales, from the rapid (~ 1 minute) variability observed during extreme, CME-induced sudden commencements such as observed on March 24, 1991 [Blake *et al.*, 1992], to time scales the order of days, as commonly seen during solar-minimum storms associated with high-speed streams [Li *et al.*, 1997]. While the wide variation in time scales suggests that different acceleration mechanisms may operate at different times, there has been increasing evidence that Pc-5 ULF oscillations, with frequencies in the 1.5-10 mHz range, play a fundamental role in storm-time particle dynamics over periods of hours and longer. In a comparison of the May 27, 1996 magnetic cloud event with that of January 10-11, 1997, Baker *et al.* [1998b] found that large-amplitude oscillations in the Pc-5 frequency range were associated with the relativistic electron event of the 1997 storm, while the 1996 storm, which did not exhibit extensive Pc-5 activity, had no comparable increase in electron fluxes. Likewise, an analysis of the May 15, 1997 electron event also showed large increases in Pc-5 activity [Baker *et al.*, 1998a]. Even more compelling evidence was reported by Rostoker *et al.* [1998], a strong correlation between Pc-5 power and relativistic electron fluxes over a 90 day period, with large increases in wave activity preceding increases in geosynchronous electron fluxes by 1-2 days.

In MHD/particle simulations of the January 1997 event

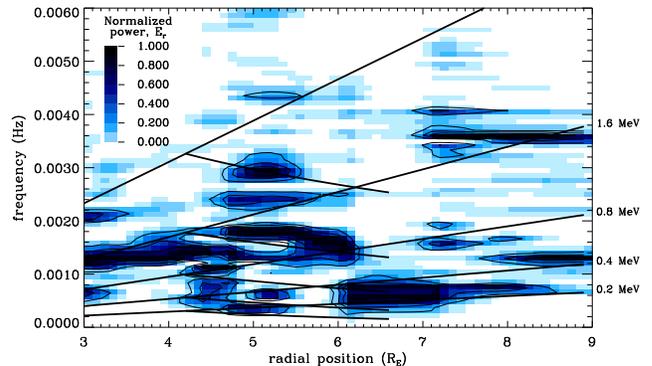


Figure 1. Power spectrum of the toroidal component of the simulated electric field in the period between 0900 and 1200 UT on January 10, 1997, at midnight MLT. The ascending black lines indicate the dipole drift frequency as a function of L for the indicated energies, while the descending lines between 4.2 and $6.6 R_E$ give the drift frequencies of particles at constant first adiabatic invariant.

[Hudson *et al.*, 1999a, b], inward radial transport and adiabatic acceleration of outer zone electrons was compared with *in situ* observations over a period of several hours. Following the observation of large-amplitude, near-monochromatic ULF oscillations in the H component of the magnetic fields at College and Gakona, Alaska during this period, a spectral analysis of the MHD fields used to drive the simulations was undertaken. The result of this analysis for the radial electric field in the equatorial plane (corresponding to Alfvénic oscillations in the H component of the magnetometer fields) is shown in Fig. 1, at the approximate local time of the Alaska magnetometers. Coherent structure appears in the radial range corresponding to the region of greatest flux increase over a 12 hour period [Reeves *et al.*, 1998], at frequencies matching the dipolar drift frequencies of electrons indicated by straight lines at constant energy (positive slope) and first adiabatic invariant (negative slope). This mode structure analysis led to the proposal that electrons could be adiabatically accelerated through a drift-resonance via interaction with toroidal-mode ULF waves [Hudson *et al.*, 1999a]. The goal of this paper is to quantitatively investigate the nature of such drift-resonant acceleration, by tracking particle trajectories in a simplified field model consisting of a compressed dipole and global, toroidal-mode Pc-5 ULF waves.

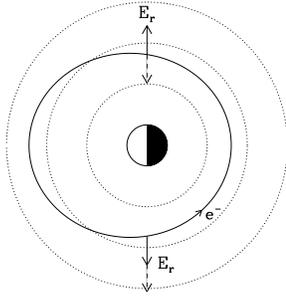


Figure 2. Sketch of an electron drift path in a compressed dipole, with electric fields indicated for an $m=2$ mode. Solid arrows indicate the electric field at $t=0$ for an electron starting at dusk, while the dashed arrows indicate the electric field direction half a drift period later.

Model and Theory

The coherent ULF mode structure evident in Fig. 1 is consistent with interpretation as toroidal-mode field line resonances. For perfectly conducting boundary conditions at the ionosphere, the fundamental mode will contain a magnetic node at the equator, while the electric field is purely radial, with ionospheric nodes and equatorial antinodes [Cummings *et al.*, 1969]. The proposed acceleration mechanism is illustrated in Fig. 2. Here an equatorially-mirroring electron in a compressed dipole interacts with a global $m=2$ toroidal-mode wave of frequency ω . At $t=0$ the electric fields are indicated by the solid arrows. An electron starting at dusk and moving with a drift frequency $\omega_d = \omega$ would first see a positive radial electric field while undergoing negative radial motion, and half a drift period later a negative electric field while moving radially outward. The resulting product $E_r dr$ is therefore negative over the orbit of the electron, leading to a net energy increase.

Based on the latitudinal structure of the fundamental toroidal mode, we begin with a simple, time-independent equatorial field model of the following form:

$$B(r, \phi) = \frac{B_0 R_E^3}{r^3} + b_1(1 + b_2 \cos \phi). \quad (1)$$

The first term represents a dipole magnetic field of strength B_0 at the surface of the Earth, while the second term models the compression of the field resulting from solar wind dynamic pressure. ϕ is taken to be zero at local noon, and constants b_1 and b_2 are selected based on measured magnetic field values. For this study, $B_0=27500$ nT, and b_1 and b_2 have been selected to give geosynchronous magnetic fields of nominal values ~ 105 nT at local noon and ~ 75 nT at local midnight. The electric field, with equatorial antinodes, is modeled as

$$E(r, \phi, t) = E_0(r, \phi) + \sum_{m=0}^{\infty} \delta E_{rm} \sin(m\phi + \omega t + \xi_m), \quad (2)$$

where the first term represents magnetospheric convection fields, while the second term is a superposition of global

toroidal field line resonances in the equatorial plane. Here m represents the azimuthal mode number of the ULF wave, ω the frequency, and δE_{rm} and ξ_m the amplitude and phase lag of the mode m .

Analytic calculations of relativistic electron interaction with the above ULF wave have been made and are similar to those given by Chan *et al.* [1989]; only the results will be given here. One finds a resonance condition

$$\omega - (m \pm 1)\omega_d = 0 \quad (3)$$

where ω_d is the azimuthal drift frequency, and an energy resonance half-width for the “ $m \pm 1$ ” resonance

$$\Delta \mathcal{E}_{m \pm 1} = \sqrt{\frac{2e\delta E_{rm}\delta r}{(m \pm 1) \left[\frac{\partial(\ln \omega_d)}{\partial \mathcal{E}} \right]_{\mathcal{E}=\mathcal{E}_{m \pm 1}}} } \quad (4)$$

where δr is a measure of the asymmetry of the drift orbit (i.e., the drift orbit in the compressed dipole field is assumed to be of the form $r(\phi) = r_0 + \delta r \cos \phi$).

Eq. (3) gives the relation between wave frequency and energy $\mathcal{E}_{m \pm 1}$ of a resonant electron on a given drift shell (for a given m value), while Eq. (4) gives the range in energy $\mathcal{E} \pm \Delta \mathcal{E}_{m \pm 1}$ over which an electron experiences resonant interaction with the wave. The ± 1 factor in Eq. (4) is due to the $m = 1$ day-night asymmetry in the compressed dipole. In Eq. (4) the numerator represents the stronger driving effect of larger wave amplitude and of larger drift-orbit asymmetry; the denominator shows the detuning effect of the energy dependence of the drift frequency. Note that for $m \geq 2$ there are two possible resonances: one for $m+1$ and one for $m-1$.

We verify the resonant interaction of particles by numerically solving the relativistic guiding-center drift equations [Northrop, 1963] for equatorially-mirroring electrons moving in the simplified field model described above. Wave frequencies are matched to particle energies by solving $\oint d\phi/\dot{\phi}$ for a given value of M , the relativistic first adiabatic invariant, along a contour of constant B while matching the resonance condition (3).

Results and Discussion

The effect of the modeled Pc-5 oscillations on a single particle can be seen in Fig. 3. Global oscillation in an $m=2$ mode is set up at a single frequency of 3 mHz and given an amplitude of 3 mV/m. A geosynchronous electron is started at local noon with an initial energy of 1.63 MeV and allowed to interact with the waves over 10 drift periods. Fig. 3(a) shows the product $E_r dr$, consistently negative, as the electron drifts around the earth. An integrated potential drop of -128 kV is experienced by the electron as it moves through its 10 drift orbits. The energy is plotted in panel 3(b), where the particle gains the 128 keV expected from panel (a). The increase in energy and conservation of the first adiabatic invariant results in transport radially inward by $\sim 0.3 R_E$.

In Fig. 3(c), a simple dawn-dusk electric field of 3 mV/m, uniform in magnitude, is added, and the total energization

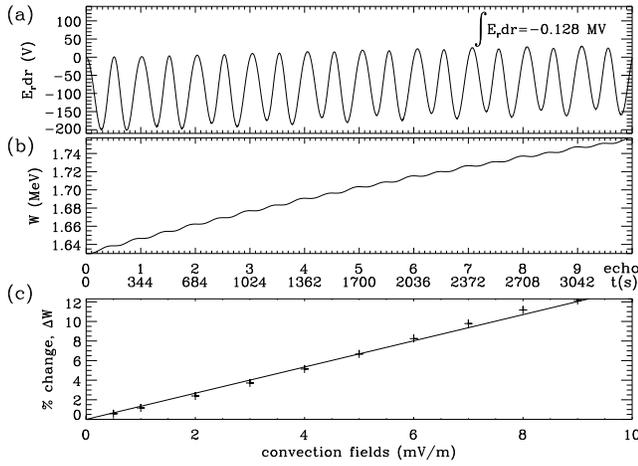


Figure 3. $E_r dr$ (a) and energy (b) as a function of time and drift echoes for an electron with initial energy of 1.63 MeV, moving in a 3 mHz, $m=2$ toroidal field of amplitude 3 mV/m. (c) Relative increase in energization for same particle, as a function of increasing dawn-dusk convection electric field.

for the same initial conditions is recorded after a set period of time. A linear increase in the rate of energization with increasing convection electric field results.

To verify the resonant nature of this acceleration, particles of different initial energies (hence different drift frequencies) can be started in a single specified mode, and maximum energy gain recorded. An example is shown in Fig. 4(a) for a 4 mHz, 3 mV/m field in an $m=2$ mode, with no convection electric field. All electrons are started at local dusk; those with energies near \mathcal{E}_1 ($\omega_d = \omega$) see the greatest increase in energy. Also, consistent with Eq. (3), a second peak is clearly evident where $\omega_d = \omega/3$, at energy \mathcal{E}_3 .

Phase space plots of the particle motion provide a useful way of examining the behavior of a driven system [e.g. *Lichtenberg & Lieberman, 1983*]. Fig. 4(b) shows a Poincaré plot consisting of the particle energy and azimuthal position recorded at increments of a wave period; the resulting plot shows the expected resonant island centered at $\mathcal{E}_1 \sim 3.14$ MeV, and three smaller islands at \mathcal{E}_3 around 850 keV, consistent with Eqs. (3) and (4). The dropout in energy gain seen at \mathcal{E}_1 in panel (a) is a result of the saddle point at $\phi_0 = 270$ degrees in panel (b). For the electrons and fields used to generate panels (a) and (b), and a radial displacement $\delta r = 0.23 R_E$ (consistent with trajectory calculations), the resonant width predicted by Eq. (4) is ~ 0.17 MeV, in good agreement with the maximum energy gain in Fig. 4(a) and primary island width in Fig. 4(b).

The effect of a convection electric field on an electron's phase plane dynamics is depicted in 4(c) for a single particle beginning at a point outside the resonant separatrix. The uniform convection field imposed transforms the primary resonant center in the reduced phase space of panel (b) into a feature resembling a stable attractor. An important implication of this result is that particles may be adiabatically accelerated from energies outside the resonance described in

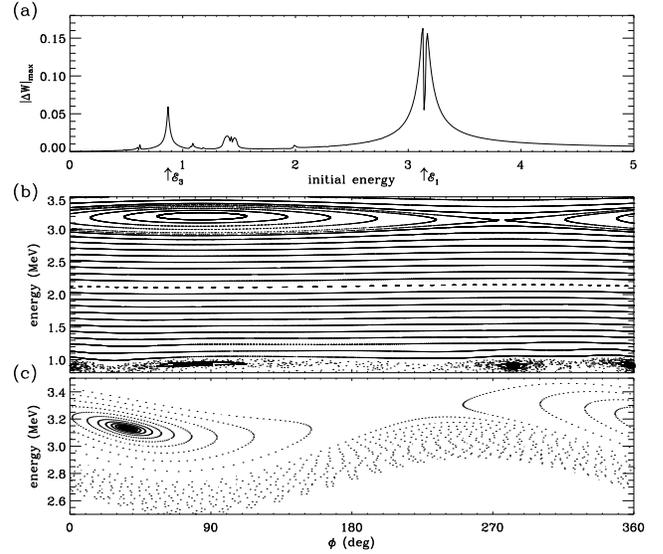


Figure 4. (a) Energy gained by electrons with different initial energies in a 3 mV/m, $m=2$ toroidal electric field oscillating at 4 mHz, each particle beginning at $5.0 R_E$ at dawn local time. \mathcal{E}_1 and \mathcal{E}_3 indicate the energies of electrons with $\omega_d = \omega$ and $\omega_d = \omega/3$, respectively. (b) Poincaré map showing the phase plane dynamics of particles with uniform first adiabatic invariant, moving in the same purely toroidal-mode fields as (a). (c) Poincaré map of a single particle starting at local dusk at 2.5 MeV moving with the same first adiabatic invariant and fields as above, with the addition of a uniform dawn-dusk convection electric field of 5 mV/m.

Eqs. (3) and (4). In principle it is possible to adiabatically accelerate electrons with 10-100 keV energies at the magnetopause to MeV energies in the inner magnetosphere, using drift-resonant acceleration and a strong convection electric field. For example, an electron with an initial energy of 80 keV at $10 R_E$ at local noon would have an energy around 200 keV at geosynchronous, and exceeding 1.1 MeV at $3 R_E$.

A second implication of the effect of the convection field is that it is possible to accelerate particles in bulk using resonance with toroidal waves. That is, without the effect of the convection fields, particles on one side of the resonance would gain energy while particles on the other side of the resonance would lose energy, resulting in a bulk acceleration limited to that arising from energy asymmetries in the resonant island. The addition of the convection electric field makes it possible to accelerate particles regardless of their initial phase. This can be seen in Fig. 5, where we have started a ring of electrons at constant L with energies just above the minimum of the resonant separatrix, $W \sim \mathcal{E}_1 - \Delta \mathcal{E}_1$, and let them evolve in time in response to 2 mHz, 3 mV/m toroidal oscillations superposed on a constant 5 mV/m convection electric field. Fig. 5(a) shows later phase bunching, resulting from the effect of radial electric fields on the azimuthal drift velocity of the particles. Average particle energy is plotted in Fig. 5(b), where it is clear that there has been net bulk energization. Note that the mag-

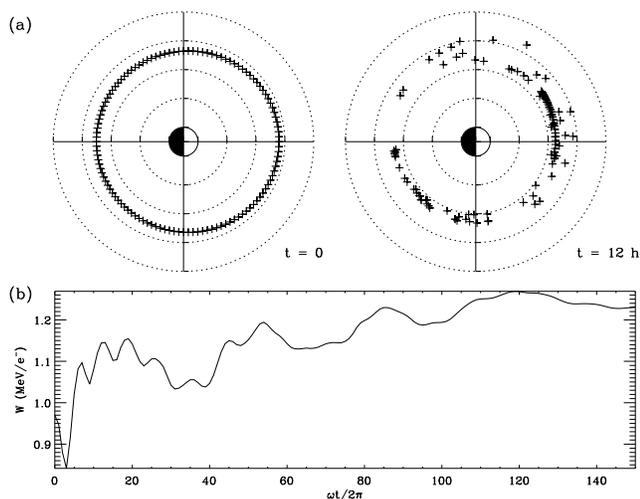


Figure 5. (a) Particle positions for a ring of near-geosynchronous particles moving in a 2 mHz, 3 mV/m toroidal oscillation with an imposed dawn-dusk convection electric field of 5 mV/m. (b) Average particle energy for particles depicted in (a), as a function of wave cycle.

nitude of the convection field affects only the rate at which electrons achieve resonant energy, as evidenced in Fig. 3(c), while the resonant energy itself is determined by Eq. (3).

Conclusion

We have shown that it is possible to adiabatically accelerate and transport magnetospheric electrons through drift-resonant interaction with toroidal-mode Pc-5 oscillations, both individually and in bulk. The rate of energization increases with both increasing radial distortion of the magnetic field, and increasing convection electric field. Both act to increase δr , the amplitude of drift-orbit asymmetry, which affects the range of resonant interaction with the wave as described by Eq. 4. The phase bunching seen in Fig. 5(a) suggests observations that may test this mechanism. Spectral analysis of lower-energy (~ 100 keV) electron fluxes seen at geosynchronous, for example, has shown variations in periods matching the electron drift frequency at discrete energies, indicating dispersionless phase bunching of electrons [Lessard and Reeves, 1999]. However, a survey of this type has yet to be undertaken for MeV electrons.

Poloidal field line resonances, with an associated E_ϕ , generally have a parallel magnetic disturbance in the equatorial plane and therefore cannot be quantitatively examined using the simple equatorial field model outlined above. However, limited investigation has been undertaken, and the results in the equatorial plane interpreted in a bounce-averaged fashion. Preliminary indications are that, while poloidal oscillations alone are able to efficiently accelerate single particles in a nonaxisymmetric background magnetic field, even without pitch angle scattering [Liu et al., 1999], the introduction of a convection field causes particles below the resonant separatrix to lose energy. This result suggests

that a necessary condition for bulk electron acceleration by a drift-resonant mechanism is that there be more power in the toroidal than poloidal modes, consistent with what has so far been seen in MHD/particle simulations of recent magnetic storms [Hudson et al., 1999a, b]. A survey correlating ULF wave polarizations with energetic electron events may provide further observational evidence for the proposed mechanism. Work is in progress to simulate bulk particle dynamics using a spectrum of poloidal and toroidal frequencies, similar to that seen in Fig. 1.

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