8.1 Electromagnetic force on a charge

A charge $q$ is at rest in an instantaneous inertial frame $K'$ in a purely electric field $E'$, so the force on $q$ (in frame $K'$) is $F' = qE'$. Suppose the charge $q$ has a velocity $v = v\hat{x}$ in frame $K$.

(a) Express the three components of $F = dp/dt$ in terms of the components of $F'$.

(b) Express the three components of $E'$ in terms of the components of $E$, $B$, $v$ and constants.

(c) Combine the results of (a) and (b) to obtain the formula for the force $F$ as a function of $E$, $B$, $v$ and constants. Please note that you must not just write down the equation for $F$ (which will probably be obvious to you from physical reasoning).

8.2 A variational principle for a (well-known) field equation

Use the variational principle

$$\delta \int L dV dt = 0$$

with

$$L = \frac{1}{2} \left[ \frac{1}{c^2} (\partial_t \psi)^2 - \nabla \psi \cdot \nabla \psi - \mu^2 \psi^2 \right],$$

where $\mu$ is a constant, to find the differential equation obeyed by $\psi(r,t)$.

8.3 The inhomogeneous Maxwell equations

Consider the action integral $S = (1/c) \int \Lambda d\Omega$, where $d\Omega = cdt dV$ and $\Lambda$ is a function of $A_i$, $A_{i,k} = \partial A_i / \partial x^k$, and $x^k$.

(a) Show that the variation $\delta S = 0$ yields the Euler-Lagrange equations

$$\frac{\partial}{\partial x^k} \left( \frac{\partial \Lambda}{\partial A_{i,k}} \right) - \frac{\partial \Lambda}{\partial A_i} = 0. \tag{1}$$

(b) For the electromagnetic field

$$\Lambda = -\frac{1}{c} j^i A_i - \frac{1}{16\pi} F^{lm} F_{lm} \tag{2}$$

Use equation (1) to obtain the manifestly-covariant field equations for this $\Lambda$, then show that they correspond to the inhomogeneous Maxwell equations in 3-vector form.

8.4 Read sections 28–33 of Landau and Lifshitz.

After you have read these sections, for credit write “I have read sections 28–33”.