6.1 The charged particle equation of motion

In the usual notation the Lagrangian for a relativistic charged particle in an electromagnetic field is

\[ L = \frac{mc^2}{\gamma} + \frac{e}{c} A \cdot v - e\phi. \]

(a) Use \( L \) to obtain the equation of motion for \( dp/dt \) in terms of \( v, E, B \), and constants, showing complete details of the calculations.

(b) Compare the result of part (a) to the corresponding pre-relativistic equation of motion and comment on the similarities and differences.

6.2 The manifestly covariant charged particle equation of motion

The motion of a particle of mass \( m \), charge \( e \), and four-velocity \( u^i \) in an electromagnetic four-potential \( A_i \) can be described by the manifestly-covariant Lagrangian

\[ \mathcal{L} = mc\sqrt{u_i u^i} + \frac{e}{c} A_i u^i, \]

where the action \( S = -\int \mathcal{L} ds \).

(a) Use \( \mathcal{L} \) and Hamilton’s Principle \( \delta S = 0 \) to obtain the corresponding manifestly covariant equation of motion for \( du_i/ds \).

(b) Show that the canonical four-momentum \( P^i = \partial \mathcal{L} / \partial u_i \) has time and space components

\[ P^i = \left( \frac{\epsilon + e\phi}{c}, p + \frac{e}{c} A \right). \]

6.3 A particle in uniform static electric and magnetic fields

A particle with charge \( q \) and mass \( m \) starts from rest and accelerates under the influence of a uniform static electric field \( E = E\hat{y} \) and a uniform static magnetic field \( B = B\hat{z} \) (where \( \hat{y} \) and \( \hat{z} \) are unit vectors in the \( y \) and \( z \) directions), with \( E > B \) (both positive).

As the particle accelerates toward infinite energy, the angle between the velocity and the \( y \)-axis approaches a constant value \( \alpha \). Derive the expression for \( \alpha \).

6.4 Read sections 17–22 of Landau and Lifshitz.

After you have read these sections, for credit write “I have read sections 17–22”.