3.1 Moving sidewalks

On their 21st birthday, one twin gets on a moving sidewalk, which carries her to star X at speed \(\frac{4}{5}c\); her twin bother stays home. When the traveling twin gets to star X, she immediately jumps onto the returning moving sidewalk and comes back to earth, again at speed \(\frac{4}{5}c\). She arrives on her 39th birthday (as determined by her watch).

(a) How old is her twin brother (who stayed at home)?

(b) How far away is star X? Give your answer in light years.

Call the earth system \(K\) (assume it is an inertial system), the outbound sidewalk system \(\bar{K}\), and the inbound one \(\tilde{K}\). All three systems set their clocks, and choose their origins, so that \(x = \bar{x} = \tilde{x} = 0\), \(t = \bar{t} = \tilde{t} = 0\) at the moment of departure.

(c) What are the coordinates \((x, t)\) of the jump (from outbound to inbound sidewalk) in \(K\)?

(d) What are the coordinates \((\bar{x}, \bar{t})\) of the jump in \(\bar{K}\)?

(e) What are the coordinates \((\tilde{x}, \tilde{t})\) of the jump in \(\tilde{K}\)?

(f) If the traveling twin wanted her watch to agree with the clock in \(\tilde{K}\), how would she have to reset it immediately after the jump? If she did this, what would her watch read when she got home? (This wouldn’t change her age, of course—she’s still 39—it would just make her watch agree with the clocks in \(\tilde{K}\).)

(g) If the traveling twin is asked the question, “How old is your brother right now?”, what is the correct reply (i) just before she makes the jump, (ii) just after she makes the jump? (Nothing dramatic happens to her brother during the split second between (i) and (ii), of course; what does change abruptly is his sister’s notion of what “right now, back home” means.)

(h) How many earth years does the return trip take? Add this to (ii) from (g) to determine how old she expects him to be at their reunion. Compare your answer to (a).

3.2 Practice with four-tensors

(a) Suppose that for any tensor components \(B_i\), the product \(C^{ij}B_j\) are tensor components. Prove that \(C^{ij}\) are tensor components (i.e., prove the quotient rule for this case).

(b) Show that \(\delta^i_k\), where \(\delta^i_k = 1\) if \(i = k\) and \(\delta^i_k = 0\) otherwise, transforms like mixed components of a tensor (the unit tensor, in this case).

(c) Show that \(g_{ij}\), defined such that \(ds^2 = g_{ij}dx^idx^j\), transforms like the covariant components of a tensor. Hint: use a variation of the quotient rule.

(d) For \(F_{ij}\) antisymmetric, show that \(F_i^j,k,F_k^j = -F_{ij,k}F^{jk}\).

(e) Show that \(d\Omega = dx^0 dx^1 dx^2 dx^3\) is invariant under Lorentz transformations.

3.3 Read sections 7 and 8 of Landau and Lifshitz.

After reading these sections, for credit write “I have read sections 7 and 8 of Landau and Lifshitz.”