1. "Brute force" numerical simulation of a plasma

The aim of this problem is to estimate the amount of computer CPU time required to numerically simulate the exact evolution of a plasma by solving the individual Lorentz-force equations of motion.

Assume the following:

- An ideal classical nonrelativistic plasma, electrostatic interactions, no external fields.
- Number densities $n_e = n_i = n$ and temperatures $T_e \approx T_i = T$.
- The CPU time required for one floating point operation is $\delta\tau = 10^{-12}$ s, 10 numerical time steps are required to resolve the fastest time scale in the system, and 10 floating point operations are required to calculate a square root.

(a) Obtain a formula for $\tau_{CPU}$, the CPU time required to numerically simulate the exact evolution of all the particles in a single Debye cube. Assume a simple numerical method such as Euler’s method for stepping the Newton-Lorentz equations of motion in time. Give your answer in terms of $\Lambda$ (the number of particles in the Debye cube) and $\delta\tau$.

(b) How much CPU time would be needed to simulate one Debye cube of a space plasma with $n=$5 cm$^{-3}$ and $T=100$ eV:

i. For one plasma period (the fastest time scale in the system)?

ii. For one electron-ion collision period (one of the longer time scales in the system)?

Use the fact that the electron-ion collision frequency $\nu_c$ is given approximately by $\nu_c/\omega_p = \ln \Lambda/2\pi\Lambda$, where $\omega_p$ is the plasma frequency.

iii. For one electron-ion collision period, and for a volume of plasma equal to the system size of about 64,000 km (approximately 10 Earth radii, roughly the size of the inner magnetosphere).

(c) Comment on your answers to part (b).

2. The relation between differential flux $j$ and phase-space density $f(x, p, t)$

In class we showed that $f = j/p^2$ for nonrelativistic particles. Show that this useful expression also holds for relativistic particles.

3. Some dimensionless parameters

(a) The basic discrete-particle parameters of a plasma are $m_s, q_s, n_s$, and $(3/2)T_s$, the mass, charge, density and mean energy per particle. From these parameters use dimensional analysis to construct a dimensionless parameter and show that it is equivalent to the plasma parameter $\Lambda_s = n_s\lambda_s^3$.

(b) Evaluate the ratio of the distance of closest approach $p_0$ to the mean interparticle distance. Express this ratio in terms of $\Lambda_s$ and interpret the result physically for a weakly-coupled plasma.

(c) At any instant the probability that an electron will be experiencing a large-angle collision with another electron in the plasma is given approximately by $P_L = n_e p_0^3$ (convince yourself of this). Express $P_L$ in terms of $\Lambda_s$ and interpret the result physically for a weakly-coupled plasma.